FINITE TEMPERATURE EFFECTIVE POTENTIAL IN THE HARTREE-FOCK APPROXIMATION

Giovanni AMELINO-CAMELIA
Center for Theoretical Physics, Laboratory for Nuclear Science,
and Department of Physics, Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, USA

and

So-Young PI
Department of Physics, Boston University, 590 Commonwealth Ave.
Boston, Massachusetts 02215, USA

ABSTRACT

In order to investigate the nature of the phase transition, we study the finite temperature effective potential for the $\lambda\Phi^4$ theory in the Hartree-Fock approximation, which sums up all the daisy and superdaisy diagrams.

1. Introduction

Temperature induced symmetry-changing phase-transitions in quantum field theory[1−3] are important ingredients in modern cosmological scenarios. The approximate critical temperature of a given phase transition can be determined by calculating the one-loop finite temperature effective potential[2]. However, cosmological scenarios often rely on the detailed nature of the phase transition. In particular, it has been recently suggested[4] that the observed baryon asymmetry might have been generated at the electroweak phase transition, if this transition is of first order, and that the rate of baryon number violation at the electroweak phase transition depends exponentially on the expectation value of the Higgs field just below the phase transition. Unfortunately, when the temperature T is near or above the critical temperature T_c the one-loop approximation does not give a reliable estimate of the finite temperature effective potential $V_T(\phi)$. The fact that when $T \geq T_c$ the one-loop term restores the symmetries which are *spontaneously broken* by the tree-level potential at T=0, tells us that at the phase transition the perturbative approach based on the ordinary loop expansion[2] breaks down. For example, in the $\lambda\Phi^4$ scalar theory the tree level potential is (in terms of renormalized quantities)

$$V^{tree} = -\frac{m_R^2}{2}\phi^2 + \frac{\lambda_R}{4!}\phi^4 \ , \tag{1}$$

and the one-loop contribution can be approximated by[2]

$$V^{one-loop} \sim \frac{\lambda_R T^2}{48}\phi^2 \ , \tag{2}$$

and therefore at high temperatures (i.e. $T \geq \sqrt{24 m_R^2/\lambda_R} \simeq T_c$) $V^{one-loop} \geq V^{tree}$ for all $\phi \leq T$. By using power counting it has been argued[3,5] that the dominant high-temperature contributions come from the infinite classes of *daisy* and *superdaisy* diagrams[2,3,5] and that they are non-negligible for all $\phi < T$ at $T \sim T_c$. Using again power counting one can show[5,6] that the improved approximation of $V_T(\phi)$ obtained by adding the daisy and superdaisy contributions to the one-loop result should be reliable (even when $T \sim T_c$) for all $\phi > gT$, where g is the (largest) coupling constant of the theory ($g{=}\sqrt{\lambda}$ in $\lambda\Phi^4$ theory), and up to order g^3.

The recent interest in temperature-induced phase transitions, due to the *electroweak baryogenesis* idea, has motivated numerous attempts[5−11] to evaluate the leading (and subleading) high-temperature contributions from daisy and superdaisy diagrams. Some of the authors have calculated an "improved one-loop" effective potential in which the tree-level propagators are replaced by temperature dependent effective propagators, which were obtained by summing the dominant high-temperature contributions from infinite-series of certain classes of self-energy graphs in perturbation theory. When one considers only the leading corrections to the effective propagators all results are in agreement with each other. However, there have been various disagreements when the subleading corrections to the effective propagators, which are important in determining the detailed nature of the phase transition, are included.

The difficulties that arise in the improved one-loop calculations are due to the fact that the substitution of improved propagators in the one-loop effective potential is an ad-hoc approximation. One needs a consistent loop expansion of the effective potential in terms of the full propagator, as the one given by the Cornwall-Jackiw-Tomboulis (CJT) formalism of the effective action and potential for composite operators[12]. Using the CJT formalism it is easy to see[13] that the daisy and superdaisy resummed effective potential (the sum of the one-loop and the leading daisy-superdaisy contributions) is given by

$$V_T(\phi) \simeq V_T^{res}(\phi, G_0) \ , \tag{3}$$

$$V_T^{res}(\phi, G) \equiv V_{cl}(\phi) + \frac{1}{2}\sum\!\!\!\!\!\!\int_k \ln G^{-1}(k) + \frac{1}{2}\sum\!\!\!\!\!\!\int_k [D^{-1}(\phi;k)G(k) - 1] + V_2^*(\phi, G) \ , \tag{4}$$

$$\left[\frac{\delta V_T^{res}(\phi, G)}{\delta G}\right]_{G=G_0} = 0 \ , \tag{5}$$

where

$$\sum\!\!\!\!\!\!\int_p \ \equiv T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \ , \tag{6}$$

and V_2^* is given by the leading two-loop contributions to the effective potential for composite operators.

We shall present the daisy and superdaisy resummed finite temperature effective potential V_T^{res} for the $\lambda\Phi^4$ theory in the imaginary time formalism. We evaluate numerically the exact V_T^{res}, and we investigate the analytic structure of V_T^{res} using an high-temperature expansion. Instead of dropping various finite and divergent terms, as has been done often in the recent literature, renormalization is carried out explicitly.

2. $\lambda\Phi^4$ Theory

2.1. Daisy and Superdaisy Resummmed Effective Potential

The Euclidean Lagrange density for the single scalar field with $\lambda\Phi^4$ interaction is given by

$$L = \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4!}\Phi^4 \ . \tag{7}$$

The tree-level propagator is

$$D(\phi;k) = \frac{1}{k^2 + m^2 + \frac{\lambda}{2}\phi^2} \ , \tag{8}$$

and the vertices of the shifted ($\Phi \to \Phi + \phi$) theory are given by

$$L_{int}(\phi;\Phi) = \frac{\lambda}{6}\phi\Phi^3 + \frac{\lambda}{4!}\Phi^4 \ . \tag{9}$$

Following Eqs.(3)-(5) one finds that the daisy and superdaisy resummed effective potential for the $\lambda\Phi^4$ theory is given by[13]

$$\begin{aligned} V_T^{res}(\phi, G) \quad &= \quad V_{cl}(\phi) + \frac{1}{2}\sum\!\!\!\!\!\!\!\int_k \ln G^{-1}(k) + \frac{1}{2}\sum\!\!\!\!\!\!\!\int_k [D^{-1}(\phi;k)G(k) - 1] \\ &\quad + \frac{3}{4!}\lambda\sum\!\!\!\!\!\!\!\int_k G(k)\sum\!\!\!\!\!\!\!\int_p G(p) \ . \end{aligned} \tag{10}$$

The last term on the r.h.s. of Eq.(10) is the contribution of the double-bubble diagram[13], which is the leading two-loop contribution to the effective potential for composite operators in $\lambda\Phi^4$ theory.

By stationarizing V_T^{res} with respect to G we obtain the gap equation:

$$G^{-1}(k) = D^{-1}(k) + \frac{\lambda}{2}\sum\!\!\!\!\!\!\!\int_p G(p) \ . \tag{11}$$

It is straightforward to show by iteration that Eq.(11) generates all daisy and superdaisy diagrams that contribute to the full propagator in ordinary perturbation theory. This is called Hartree-Fock approximation[12].

It is convenient to take the following *Ansatz* for $G(k)$

$$G(k) = \frac{1}{k^2 + M^2} \ . \tag{12}$$

In Eq.(12) we have made no assumption on the form of $G(k)$; in fact, at this stage the "effective mass" M is an unknown function of the momentum k to be determined using Eq.(11). Substituting Eqs.(8) and (12) in Eq.(11), one obtains

$$M^2 = m^2 - \frac{\lambda}{2}\phi^2 - \frac{\lambda}{2}P(M) \ , \tag{13}$$

where

$$P(M) \equiv \sum\!\!\!\!\!\!\int_k \frac{1}{k^2 + M^2} \ , \tag{14}$$

which implies that in this approximation M is momentum independent.

In terms of the solution $M(\phi)$ of Eq.(13), the daisy and superdaisy resummed effective potential takes the form

$$V_T^{res}(\phi, M(\phi)) = V^0 + V^I + V^{II} \ , \tag{15}$$

$$V^0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \ , \tag{16}$$

$$V^I = \frac{1}{2}\sum\!\!\!\!\!\!\int_k \ln[k^2 + M^2(\phi)] \ , \tag{17}$$

$$V^{II} = -\frac{\lambda}{8}P(M(\phi))P(M(\phi)) \ , \tag{18}$$

where V^0, V^I, and V^{II} are the classical, one-loop, and two-loop contributions respectively.

2.2. Renormalizing the Effective Potential

The expression of $V_T^{res}(\phi, M(\phi))$ in (15) contains divergent integrals. Moreover, due to the fact that our approximation is self-consistent, reflecting the non-linearity of the full theory, $M(\phi)$, the argument of V_T^{res}, is not well-defined because of the infinities in $P(M)$. We shall first obtain a well-defined, finite expression for $M(\phi)$ by a renormalization.

We define renormalized parameters m_R and λ_R as

$$\pm\frac{m_R^2}{\lambda_R} = \frac{m^2}{\lambda} + \frac{1}{2}I_1 \ , \tag{19}$$

$$\frac{1}{\lambda_R} = \frac{1}{\lambda} + \frac{1}{2}I_2(\mu) \ , \tag{20}$$

where $m_R^2 > 0$, and $I_{1,2}$ are divergent integrals

$$I_1 \equiv \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2|\mathbf{k}|} = \lim_{\Lambda\to\infty}\frac{\Lambda^2}{8\pi^2} \ , \tag{21}$$

$$I_2(\mu) \equiv \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2|\mathbf{k}|} - \frac{1}{2\sqrt{|\mathbf{k}|^2+\mu^2}}\right] = \lim_{\Lambda\to\infty} \frac{1}{16\pi^2} \ln\frac{\Lambda^2}{\mu^2} \ . \tag{22}$$

μ is the renormalization scale and Λ is the ultraviolet momentum cut-off. In the following we shall choose the negative sign in Eq.(19), which allows spontaneous symmetry breaking.

When the sum on n is carried out as in Ref.2, $P(M)$ takes the form

$$\begin{aligned} P(M) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} + \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k(exp[\beta\omega_k]-1)} \\ &\equiv P_f(M) + I_1 - M^2 I_2(\mu) \ , \end{aligned} \tag{23}$$

where $\omega_k \equiv [|\mathbf{k}|^2 + M^2]^{1/2}$ and $P_f(M)$ is the finite part of $P(M)$, given by

$$P_f(M) \equiv \frac{M^2}{16\pi^2} \ln\frac{M^2}{\mu^2} + \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k(exp[\beta\omega_k]-1)} \ . \tag{24}$$

In the limit $T=0$, the first term in $P_f(M)$ survives, but the second term vanishes.

It is straightforward to see that M is finite and cut-off independent in terms of m_R and λ_R

$$M^2 = -m_R^2 + \frac{\lambda_R}{2}\phi^2 + \frac{\lambda_R}{2} P_f(M) \equiv \tilde{m}^2(\phi) + \frac{\lambda_R}{2} P_f(M) \ , \tag{25}$$

where we also defined, for later convenience, the tree-level effective mass $\tilde{m}(\phi)$.

With this finite M, we are ready to discuss the divergences in $V_T^{res}(\phi, M)$. First, carrying out the sum on n in V^I, we obtain the familiar one-loop finite temperature formula[2]

$$\begin{aligned} V^I(M) &= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} \omega_k + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3} \ln(1 - exp[\beta\omega_k]) \\ &= \frac{M^4}{64\pi^2}\left[\ln\frac{M^2}{\mu^2} - \frac{1}{2}\right] + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3} \ln(1 - exp[\beta\omega_k]) - \frac{M^4}{4} I_2(\mu) + \frac{M^2}{2} I_1 \ . \end{aligned} \tag{26}$$

At $T=0$ the first term of V^I survives and provides the zero-temperature one-loop contribution, and the second term vanishes. The last two terms are the divergence in V^I.

Divergences in the two-loop contribution V^{II} come from the square of $P(M)$. Finiteness of V_T^{res} can be shown by first combining V^0 and V^{II} using the unrenormalized form of the gap equation. When the combined expression is written in terms of renormalized parameters, the remaining divergent integrals are cancelled by those of V^I in (26). This is another indication that the two-loop contribution must be included for a finite self-consistent approximation. The resulting finite expression for V_T^{res} is

$$V_T^{res}(\phi, M(\phi)) = (V_R^0 + V_R^{II}) + V_R^I \ , \tag{27}$$

$$V_R^0 + V_R^{II} = \frac{M^4}{2\lambda_R} - \frac{1}{2}M^2 P_f(M) - \frac{\lambda}{12}\phi^4 \ , \tag{28}$$

$$V_R^I = \frac{M^4}{64\pi^2}[\ln\frac{M^2}{\mu^2} - \frac{1}{2}] + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3} \ \ln(1 - exp[\beta\omega_k]) \ . \tag{29}$$

[A constant term $m^4/(2\lambda)$ has been adjusted to obtain Eqs.(27)-(29) from Eq.(15).] In order to compare the finite temperature effective potential with and without the two-loop contribution in our later discussion, we still have to extract V_R^{II} from (28). Observing that V_R^0 should be a function of ϕ only, and that in our approximation V_R^{II} does not depend on ϕ explicitly [since the double-bubble graph does not involve any vertices that depend on ϕ] one obtains, by using the renormalized gap equation,

$$V_R^0 + V_R^{II} = \left[\frac{\lambda_R}{8}\left(\phi^2 - 2\frac{m_R^2}{\lambda_R}\right) - \frac{\lambda}{12}\phi^4\right] - \frac{\lambda_R}{8}P_f(M)P_f(M) \ . \tag{30}$$

Clearly the last term in Eq.(30) is the two-loop contribution. The quantity in the brackets is the classical contribution after renormalization is carried out. It is cut-off dependent because of the term $-\lambda\phi^4/12$, which did not get renormalized due to the structure of the gap equation. But the renormalization prescription (19)-(20) tells us that if λ_R is held fixed as $\Lambda \to \infty$, λ approaches 0_-. [A necessary condition in a non-trivial ($\lambda_R > 0$) renormalized $\lambda\phi^4$ theory is $\lambda < 0$.] As shown in large N studies[14], such theory is intrinsically unstable. On the other hand, holding $\lambda > 0$ implies $\lambda_R \to 0$ as $\Lambda \to \infty$. For $\lambda > 0$, a sensible theory can be obtained for a fixed small $\lambda_R > 0$ as an effective low energy theory, if Λ is kept fixed at a large but finite value. Such theory requires

$$\frac{\lambda_R}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2} < 1 \ , \tag{31}$$

in order to have $\lambda > 0$, and all momenta, temperature and any other physical mass scale must be much smaller than Λ. We shall consider such an effective theory.

As shown in Fig.1, the zero-temperature phase structure of the effective theory with finite Λ is similar to that of perturbation theory: there exists a minimum at a non-zero value of ϕ.

The numerical result for the effective potential in Eq.(27) (with finite Λ) at the critical temperature is reported in Fig.2. Notice that at the critical temperature the daisy and superdaisy resummed effective potential has two degenerate minima. However, the symmetry breaking minimum $\phi = \phi_b$ is very close to the symmetric minimum $\phi = 0$; in fact, $\phi_b/T_c << \sqrt{\lambda_R}$. As discussed in the introduction, the daisy and superdaisy resummed effective potential is expected to give a reliable approximation of the full effective potential only for $\phi > \sqrt{\lambda_R}T$, and therefore, since ϕ_b is located in the region that is unreliably investigated by V_T^{res}, the correct conclusion to be drawn from Fig.2 is that the phase transition of the $\lambda\Phi^4$ theory is second order or very weakly first order. Recent numerical investigations seem to indicate that this phase transition is second order.

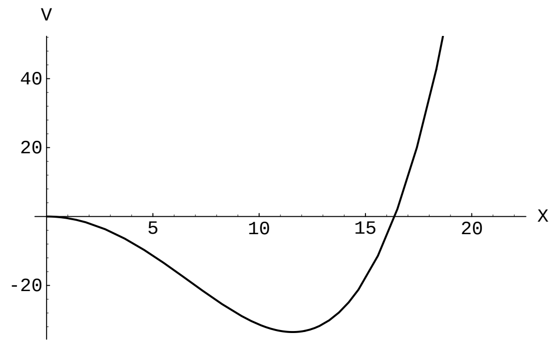

Figure 1: The daisy and superdaisy resummed effective potential at $T{=}0$. In figure $V(X) \equiv Re[V_T^{res}(X) - V_T^{res}(0)]/m_R^4$, $X \equiv \phi/m_R$, $\lambda_R{=}0.05$, $\mu{=}m_R$, and $\ln(\Lambda^2/m_R^2) = 16\pi^2$.

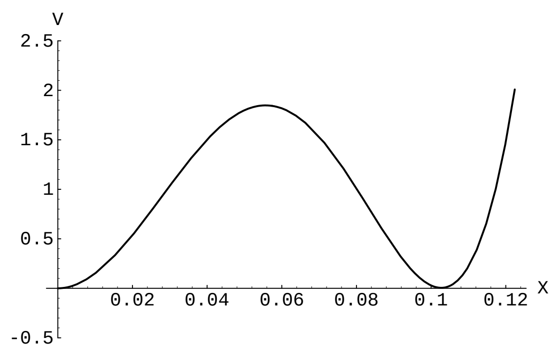

Figure 2: The daisy and superdaisy resummed finite temperature effective potential. In figure $V(X) \equiv 10^{11} Re[V_T^{res}(X) - V_T^{res}(0)]/T^4$, $X \equiv \phi/\lambda_R^{1/2} T$, $T \simeq T_c \simeq 21.920 m_R$, $\lambda_R = 0.05$, $\mu = m_R$, and $\ln(\Lambda^2/m_R^2) = 16\pi^2$.

2.3. High Temperature Approximation

Our main interest is in the form of the effective potential at high temperature (of the order of the critical temperature). At high temperature one can derive an analytic approximation of the daisy and superdaisy resummed effective potential obtained in Eq.(27) by assuming that $M^2/T^2 << 1$ in the integral expressions of $P_f(M)$ and $V_R^I(M)$.

As shown in Ref.2, for $M^2/T^2 << 1$

$$P_f(M) \simeq T^2 \left[\frac{1}{12} - \frac{1}{4\pi} \frac{M}{T} \right] . \tag{32}$$

Then the high-temperature gap equation takes the form

$$M^2 \simeq \tilde{m}^2(\phi) + \frac{\lambda_R}{24}T^2 - \frac{\lambda_R}{8\pi}MT \ . \tag{33}$$

From the solution of this equation one finds that for a small coupling $\lambda_R << 1$, the condition $M^2/T^2 << 1$ is consistent with $\tilde{m}^2(\phi)/T^2 << 1$, which is exactly the required condition for the high-temperature expansion of the perturbative calculation in Ref.2.

Now we return to Eqs.(27)-(30). Using Eq.(33) and the high-temperature expansion of $V_R^I(M)$ derived in Ref.2, we obtain the desired high-temperature analytic approximation of the daisy and superdaisy resummed effective potential:

$$V_T^{res}(\phi, M(\phi)) = V_R^0 + V_R^I + V_R^{II} \ , \tag{34}$$

$$V^0 = \frac{\lambda_R}{8}\left[\phi^2 - 2\frac{m_R^2}{\lambda_R}\right]^2 - \frac{\lambda}{12}\phi^4 \ , \tag{35}$$

$$V^I \simeq -\frac{\pi^2}{90}T^4 + \frac{M^2T^2}{24} - \frac{M^3T}{12\pi} \ , \tag{36}$$

$$V^{II} \simeq -\frac{\lambda_R}{8}\left[\frac{T^4}{144} - \frac{MT^3}{24\pi} + \frac{M^2T^2}{16\pi^2}\right] \ , \tag{37}$$

where M is analytic solution of the quadratic equation (33).

In Fig.3, the effective potential of Eqs.(34)-(37) is shown at the critical temperature. Note that the critical temperature that one obtains from the high-temperature approximation of V_T^{res} is extremely close to the one obtained from the (numerical) exact evaluation of V_T^{res}. However, the high-temperature approximation leads to a determination of the symmetry breaking minimum ϕ_b which differs by approximately 20%.

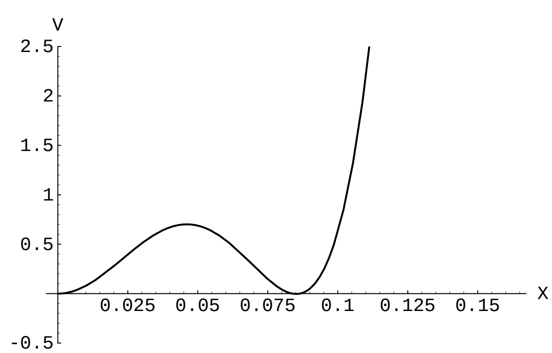

Figure 3: The high-temperature approximation of the daisy and superdaisy resummed effective potential. In figure $V(X) \equiv 10^{11} Re[V_T^{res}(X) - V_T^{res}(0)]/T^4$, $X \equiv \phi/\lambda_R^{1/2}T$, $T \simeq T_c \simeq 21.919 m_R$, $\lambda_R = 0.05$, $\mu = m_R$, and $\ln(\Lambda^2/m_R^2) = 16\pi^2$.

3. Analysis of the Results

In order to investigate the structure of the high-temperature effective potential in our approximation, we shall first consider the non-linear aspects of the high-temperature gap equation (33), which implies that $M(\phi)$ can be expanded for small λ_R as

$$M(\phi) = M_L(\phi)\Big\{1 - \frac{\lambda_R T}{16\pi M_L(\phi)} + O\Big[\Big(\frac{\lambda_R T}{16\pi M_L(\phi)}\Big)^2\Big]\Big\}\ , \tag{38}$$

where

$$M_L(\phi) \equiv \sqrt{\tilde{m}^2(\phi) + \frac{\lambda_R}{24}T^2} \tag{39}$$

solves the linearized high-temperature gap equation, i.e. (33) without the last term on the r.h.s..

The one-loop contribution V_R^I in Eq.(36) can be written as

$$V_R^I(\phi) \simeq -\frac{\pi^2}{90}T^4 + \frac{M_L^2(\phi)T^2}{24} - \frac{\lambda_R}{192\pi}M(\phi)T^3 - \frac{M^3(\phi)T}{12\pi}\ . \tag{40}$$

The term linear in M, namely the third term on the r.h.s. of Eq.(40), arises from the non-linearity of the gap equation, i.e. from the last term on the r.h.s. of Eq.(33). If we were to use the linearized gap equation, without this term, the first non-trivial correction to the perturbative one-loop effective potential would be given by the term cubic in M. However, at high temperatures the leading non-linear correction is of the same order as the term cubic in M in Eq.(40); in fact, from Eq.(33) we have

$$\Big(\frac{\lambda_R}{192\pi}M(\phi)T^3\Big)\Big/\Big(\frac{M^3(\phi)T}{12\pi}\Big) \simeq \frac{288}{192} \sim O(1) \tag{41}$$

for $T >> \phi$. When one includes the two-loop contribution given in Eq.(37) the MT^3 term disappears and the high temperature daisy and superdaisy resummed effective potential in our approximation is (neglecting some ϕ-independent contributions)

$$V_T^{res}(\phi) = V_R^0(\phi) + \Big(\frac{T^2}{24}M_L^2(\phi) - \frac{T}{12\pi}M_L^3(\phi)\Big)\Big(1 + O(\lambda_R)\Big) + O(\frac{M^4}{64\pi^2}\ln T)\ . \tag{42}$$

The above analysis of our consistent approximation shows that improving the perturbative one-loop effective potential $V^I(\tilde{m})$ using the non-linear gap equation clearly leads to an erroneous result, and one must use a self-consistent method which relates the effective potential and the gap equation. However, we also find that, due to the cancellation of the leading non-linear effect, one can obtain the leading daisy and superdaisy correction by improving the one-loop with a mass $M_L(\phi)$, which is the solution of the linearized gap equation. Such a procedure, which uses the effective mass squared shifted by a ϕ-independent amount proportional to T^2, was first suggested by Weinberg[3] and later further studied by others[8,15].

If one is also interested in the subleading daisy and superdaisy correction, a consistent result can only be obtained using the composite operator method that we discussed.

Acknowledgements

This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract #DE-FG02-91ER40676. G. A.-C. is supported by the Istituto Nazionale di Fisica Nucleare, Frascati, Italy.

References

1. D. Kirzhnits, *JETP Lett.* **15** (1972) 529; D. Kirzhnits and A. Linde, *Phys. Lett.* **B42** (1972) 471; D. Kirzhnits and A. Linde, *JETP Lett.* **40** (1974) 628.
2. L. Dolan and R. Jackiw, *Phys. Rev.* **D9** (1974) 3320.
3. S. Weinberg, *Phys. Rev.* **D9** (1974) 3357.
4. For recent reviews, see: M. Dine, SCIPP 92-21; A.G. Cohen, D.B. Kaplan, and A.E. Nelson, UCSD 93-2/BU-HEP-93-4.
5. S.D.H. Hsu, in *Proceedings of the Yale/Texas Workshop on (B+L) Violation* (World Scientific); C.G. Boyd, D.E. Brahm and S.D.H. Hsu, CALT-68-1795/HUTP-92-A027/EFI-92-22 (1992).
6. M. Dine, R.G. Leigh, P. Huet, A. Linde, and D. Linde, *Phys. Rev.* **D46** (1992) 550; *Phys. Lett.* **B283** (1992) 319.
7. M.E. Shaposhnikov, *Phys. Lett.* **B277** (1992) 324.
8. M.E. Carrington, *Phys. Rev.* **D45** (1992) 2933.
9. P. Arnold, *Phys. Rev.* **D46** (1992) 2628.
10. V. Jain, *Nucl. Phys.* **B394** (1993) 707.
11. J.R. Espinosa, M. Quiros, and F. Zwirner, *Phys. Lett.* **B291** (1992) 115.
12. J.M. Cornwall, R. Jackiw, and E. Tomboulis, *Phys. Rev.* **D10** (1974) 2428.
13. G. Amelino-Camelia and S.-Y. Pi, *Phys. Rev.* **D47** (1993) 2356.
14. See, for example, S. Coleman, R. Jackiw, and H.D. Politzer, *Phys. Rev.* **D10** (1974) 2491; W.A. Bardeen and Moshe Moshe, *Phys. Rev.* **D28** (1983) 1372.
15. P. Fendley, *Phys. Lett.* **B196** (1987) 175.